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The 2008–2009 subprime credit crisis and subsequent euro sovereign debt crisis had a devastating impact on defined benefit (DB) pension plans in countries such as the United States, the United Kingdom, and the Netherlands. Funding ratios, at their lowest point since 2007, had fallen by an average of almost 40% and initial overfunding turned into serious underfunding. Funding ratios have since recovered, but in mid-2015 they were still well below the levels at the end of 2007, as shown in Exhibit 1.

Deteriorating funding ratios, tougher regulations,¹ and a more difficult business climate eroding companies' financial health led many plan sponsors to phase out their DB plans. As a result, interest in defined contribution (DC) and/or hybrid pension schemes, which imply a transfer of risk from employers to employees and retirees, has been growing rapidly. Sponsors' fear of being forced to make additional contributions to repair an underfunded status lies at the heart of such changes. It is in this context that we want to emphasize the importance of liability-driven investment (LDI) approaches to managing pension funds. An effective LDI approach is inevitably interwoven with effective risk management, and we believe that pension funds still need to

make significant progress in this area. In a recent survey Badaoui et al. [2014] found that although pension funds and sponsor companies, most from Europe, were familiar with LDI, the rate of adoption remained limited and many were still far more concerned with standalone performance than with risk management.

This article makes a contribution to the area of risk management by focusing on funding-ratio risk and proposing a framework to yield insights into the main components of funding-ratio risk. We have three main objectives. The first is to show the advantages of using funding-ratio risk, which is the reference for most practitioners and regulators, instead of surplus risk. We argue that surplus risk offers a less complete view of the health of the pension fund, and at times it can even be misleading. In an example, we show that an increase in surplus can be accompanied by a fall in the funding ratio, which leaves no doubt about the deterioration of the pension fund's financial health.

Our second objective is to propose an analytical framework for decomposing funding-ratio risk based on a standard linear factor model. Asset-only applications of such factor models are frequently used by academics and practitioners alike—for example,

EXHIBIT 1

Funding-Ratio Developments for Typical DB Pension Funds in the U.S., U.K., and Netherlands

Country	Funding Ratio End-2007	Funding Ratio End-2008	Funding-Ratio Bottom		Funding Ratio Mid-2015
			Since 2007	in Quarter	
U.S.A. ^a	113	78 (–31%)	71 (–37%)	Q3 2011	87 (–23%)
U.K. ^b	119	94 (–21%)	78 (–35%)	Q2 2012 & Q1 2015	85 (–29%)
Netherlands ^c	144	95 (–34%)	85 (–41%)	Q2 2012	99 (–31%)
Mean	125	89 (–29%)	78 (–38%)	/	90 (–28%)

Notes: Percentage changes since end-2007 are given in italics in parentheses.

^aU.S.A. based on the “Pension Fund Fitness Tracker” covering 500 public companies sponsoring large defined benefit plans. Source: UBS Global Asset Management [2015].

^bU.K. based on the “PPI 7800 index” covering 6,057 (mid-2015) defined benefit schemes. Source: Pension Protection Fund [2015].

^cNetherlands: based on the “Pensioen Thermometer” (Pension Thermometer), which focuses on the average defined benefit pension fund defined on the basis of currently around 400 funds; liabilities are here discounted with the spot swap curve. Source: AON Hewitt [2015].

the Fama–French [1992] three-factor model or the Carhart [1997] four-factor model for equity portfolios. Similar models are also available for other major asset classes.² Such factor models have been used for risk adjusting the realized performances of actively managed portfolios and absolute return strategies. Risk decompositions across factors have been performed for asset-only portfolios in several recent papers (e.g., Meucci [2009], Lohre, Neugebauer, and Zimmer [2012], Roncalli and Weisang [2012], Deguest, Martellini, and Meucci [2013], and Meucci, Santangelo, and Deguest [2015]).

There are extensions to an LDI context in which the quantity of interest is the value of assets relative to liabilities (e.g., Ransenberg, Hodges, and Hunt [2012]), but these provide a decomposition of surplus risk, not of funding-ratio risk. As far as we know, we are the first to introduce an explicit mathematical decomposition for the volatility of the funding ratio. Moreover, the proposed decomposition has the advantages of being model-free and simple to compute;³ it also includes a special *mismatch* factor to measure the sensitivity of the funding ratio to changes in the value of liabilities.

Our third objective is to illustrate how to use the framework, while addressing practical issues such as the estimation of the factor exposures. Although all the literature we are aware of uses historical observations, we propose using Monte Carlo simulations to estimate the risk factor contributions to future volatility because it avoids relying on historical moments of asset returns and funding ratio. We suggest using a two-step ordinary

least squares (OLS) approach for estimating the factor exposures. The exposure to the novel mismatch factor is estimated in the first step, and the other factor exposures are estimated in a second step via a constrained regression. Finally, we illustrate the differences between an *assets*-centric and a so-called *LHP*-centric risk decomposition (with LHP standing for liability-hedging portfolio) and explain our preference for an *assets*-centric approach.

FUNDING RATIO AND SURPLUS⁴

While pension fund practitioners and regulators focus on the *funding ratio*, defined as *assets divided by liabilities*, most academic papers and risk-reporting software have been concerned with the *surplus*, defined as *assets minus liabilities*. Authors and developers may have been thinking that the linear relationship between surplus and its constituent parts makes it easier to analyze the statistical consequences of different investment strategies, unlike a funding-ratio approach, which is relative in its constituents.

Surplus

The literature includes ample examples of the surplus, or linear, school. Sharpe and Tint [1990] and Leibowitz, Kogelman, and Bader [1992] were among the first to consider the risk–return trade-off in terms of surplus return. Realizing that the initial surplus can be zero or close to zero, implying an undefined or very

small or big surplus return, these authors proposed the ratio of the change in surplus value to the initial value of either the assets or liabilities. Sharpe and Tint opted for the assets-centric definition of surplus return, whereas Leibowitz, Kogelman, and Bader worked with its liabilities-centric form.

In terms of surplus return expectation and variance, there is little difference between the two definitions. To demonstrate this, we let A_j , L_j , FR_j , and S_j denote the values of, respectively, the assets, liabilities, funding ratio, and surplus at time j , and let R denote the one-period-ahead return, with the subscripts indicating which return is meant. In the case of a surplus return, the superscripts *ac* and *lc* are used to indicate an *assets-centric* or *liabilities-centric* definition. The assets-centric surplus return for Period 1, which runs from time 0 to 1, is then given by the following:

$$R_S^{ac} = \frac{S_1 - S_0}{A_0} = \frac{(A_1 - L_1) - (A_0 - L_0)}{A_0} \rightarrow$$

$$R_S^{ac} = R_A - \frac{1}{FR_0} \cdot R_L \quad (1)$$

and the liabilities-centric surplus return is

$$R_S^{lc} = \frac{S_1 - S_0}{L_0} = \frac{(A_1 - L_1) - (A_0 - L_0)}{L_0} \rightarrow$$

$$R_S^{lc} = FR_0 \cdot R_A - R_L \quad (2)$$

Both definitions imply different sizes for the surplus returns (note that $R_S^{lc} = FR_0 \cdot R_S^{ac}$) and imply a linear combination between the random variables R_A and R_L . Both are a kind of adjusted linear excess return of assets over liabilities, where FR_0 adjusts the returns when assets and liabilities are of different size. This linearity opens the way for the mean–variance optimization of the surplus.

Given its analytical tractability, it is not so surprising that many authors work with surplus returns. Scherer [2002], Waring [2004a, 2004b], Coutts and Fleming [2007], Monfort [2008], Waring and Whitney [2009], Ransenberg, Hodges, and Hunt [2012] and Ang, Chen, and Sundaresan [2013] are recent examples. Surplus is also used in risk software. For example, MSCI Barra expanded its BarraOne risk system to include an estimation of the value at risk (VaR) of the surplus, breaking it down into separate risk factor components

(see MSCI Barra [2013]). A similar approach is used by Legal & General's Prism system [2013a, 2013b], which is based on the PFaroe software of RiskFirst (formerly known as PensionsFirst).

Funding Ratio

The relative or funding-ratio school finds fewer advocates. Leibowitz, Kogelman, and Bader [1994] work with the concept of *funding-ratio return*—that is, with $R_{FR} = (FR_1/FR_0) - 1$. They assume that asset and liability returns are lognormally distributed. The funding-ratio return is then also lognormally distributed with parameters that can be analytically derived from the underlying lognormal distributions. In turn, Swierstra [2011] works with the *funding ratio*—that is, FR_1 itself, showing that this can be approximated via a second-order Taylor series expansion that makes it possible to express its expectation and variance as a nonlinear function of the first two moments of R_A and R_L and of their covariance.⁵ The differences between using the funding-ratio return or the funding ratio are of minor importance because the random variables R_{FR} and FR_1 are linearly related. One should mention that R_{FR} can be written as $(1 + R_A)/(1 + R_L) - 1$, as a relative and unweighted excess return of R_A over R_L . Other advocates of the relative or funding-ratio school include Hoevenaars [2008] and Martellini and Milhau [2009], with the latter explaining that the funding ratio is equivalent to the net wealth of the pension fund with total assets expressed in terms of number of units of total liabilities.

Surplus versus Funding Ratio

We can highlight three main differences between the linear and relative schools. First, the funding ratio is more commonly used by practitioners and regulators. Second, there are fundamental differences between the two: The funding-ratio return is defined as a *relative and unweighted* excess return of R_A versus R_L and it is *independent* of the initial funding-ratio level FR_0 ; whereas the surplus return is defined as a *linear and weighted* excess return of R_A versus R_L that is *dependent* on FR_0 , because FR_0 determines the relative importance of R_A and R_L . Third, the funding ratio (and funding-ratio return) is related to the net wealth expressed in units of liability values. A pension fund with a funding ratio of 2 is, in terms of its liabilities coverage, twice as wealthy as any

EXHIBIT 2

Example of Conflicting Signals of Surplus and Funding-Ratio Returns

	Assets	Liabilities	Surplus	Funding Ratio
Start Value	120.0	90.0	30.0	133.3%
End Value	132.0	100.8	31.2	131.0%
Returns				
Normal	10%	12%	4.0%	-1.8%
Assets-Centric			1.0%	
Liabilities-Centric			1.3%	

Notes: Normal is the percentage change in value divided by the starting value. Assets-centric is the percentage change in surplus divided by the starting value of assets. Liabilities-centric is the percentage change in surplus divided by the starting value of liabilities.

other pension fund with a funding ratio of 1, irrespective of the absolute size of each fund's liabilities. Such comparisons are not possible on the basis of surplus alone, which only distinguishes the difference between assets and liabilities while ignoring the scale of pension fund's liabilities.

Illustrating this last point, Exhibit 2 focuses on the evolution of a single fund through time rather than on the cross section of funds at a given moment, showing various definitions of surplus returns for a given pension fund varying between +1.0% to +4.0%. They have one thing in common: all three signal an improvement. In contrast, the funding-ratio return signals a deterioration of -1.8%. The two concepts can thus generate fundamentally different signals about the evolution of the funding status. Consequently, we focus on funding-ratio risk rather than surplus risk in this article.

RISK METHODOLOGY

Factor Model

Here we propose a framework for decomposing funding-ratio risk. The following equation relates the funding ratio at the end of Period 1 to the current and known level of the funding ratio and the next period return of assets and of liabilities:

$$FR_1 = \frac{A_1}{L_1} = \frac{A_0 \cdot (1 + R_A)}{L_0 \cdot (1 + R_L)} \rightarrow$$

$$FR_1 = FR_0 \cdot \frac{(1 + R_A)}{(1 + R_L)} \quad (3)$$

We assume that the next period assets return R_A is a linear function of the next period return of a perfect LHP—which is thus identical to the next period liabilities return R_L —and other factors. The other factors, given by F_j , with $j = 2, \dots, k$, are arbitrary for now; we discuss their choice later in the case study. The assets return can be written as

$$R_A = \beta_0 + \beta_1 \cdot R_L + \beta_2 \cdot F_2 + \dots + \beta_k \cdot F_k + u \quad (4)$$

where u is the residual return with an expectation of zero, a homoskedastic standard deviation and zero correlation with all the factors. The coefficient β_0 is the constant and β_1 to β_k are the factor loadings.

By adding 1 to both sides of Equation 4 and dividing left and right by $(1 + R_L)$, we get

$$\frac{(1 + R_A)}{(1 + R_L)} = \frac{1}{(1 + R_L)} + \beta_1 \cdot \frac{R_L}{(1 + R_L)} + \beta_2 \cdot \frac{F_2}{(1 + R_L)} + \dots + \beta_k \cdot \frac{F_k}{(1 + R_L)} + \frac{(u + \beta_0)}{(1 + R_L)} \quad (5)$$

Using the fact that $1/(1 + R_L) = 1 - R_L/(1 + R_L)$, we find

$$\frac{(1 + R_A)}{(1 + R_L)} = 1 + (\beta_1 - 1) \cdot \frac{R_L}{(1 + R_L)} + \beta_2 \cdot \frac{F_2}{(1 + R_L)} + \dots + \beta_k \cdot \frac{F_k}{(1 + R_L)} + \frac{(u + \beta_0)}{(1 + R_L)} \quad (6)$$

In what follows, we use $*$ to indicate the rescaling of variables by $1/(1 + R_L)$. Combining Equations 3 and 6, we now get

$$FR_1 = FR_0 \cdot [1 + (\beta_1 - 1) \cdot R_L^* + \beta_2 \cdot F_2^* + \dots + \beta_k \cdot F_k^* + (u + \beta_0)^*]$$

$$= FR_0 + (\gamma_1 - FR_0) \cdot R_L^* + \gamma_2 \cdot F_2^* + \dots + \gamma_k \cdot F_k^* + e \quad (7)$$

where $\gamma_j = FR_0 \cdot \beta_j$ for $j = 1, \dots, k$ and where $e = FR_0 \cdot (u + \beta_0)^*$.

Equation 7 specifies the factor model for the funding ratio. The residual e has a nonzero expected value and mild nonzero correlations with the factors. The latter is

due to the rescaling by $1/(1 + R_L)$ of the original factors and residual term in Equation 4. The factor loadings γ_j for factors 2 to k are equal to those in Equation 4 multiplied by the initial funding ratio. This adjustment is intuitive and is required when the initial funding ratio is not equal to 1.

The factor loading $(\gamma_1 - FR_0)$ for the first factor, R_L^* , is of a different nature. The volatility of the funding ratio is zero only if all assets are invested in a perfect LHP fully replicating the liabilities. In such a case, the funding ratio remains constant over time because this is equivalent to an effective interest rate hedge of size FR_0 times 100%. In practice, interest rate risks are not fully hedged and an interest rate hedge mismatch risk shows up. Denoting the interest rate hedge as IRH, we can show that the factor loading for R_L^* in Equation 7 is given by⁶

$$\gamma_1 - FR_0 = \text{IRH} - FR_0 \quad (8)$$

The right-hand side measures the size of the mismatch. We thus refer to R_L^* as the *IRH mismatch risk factor*. The mismatch must be measured by comparing IRH with FR_0 . In essence, Equation 8 shows that the factor loading of the rescaled liabilities return R_L^* in Equation 7 measures the size of the mismatch of the IRH. This mismatch contributes to the risk of the future funding ratio for as long as it deviates from zero.

Decomposing Funding-Ratio Risk

The classical approach to decomposing the risk of FR_1 would be variance decomposition. However, Fields [2003] shows that for a linear model such as that in Equation 7, the same relative contributions are obtained if one decomposes the standard deviation instead of the variance. For practitioners, the latter is probably more appealing than variance decomposition. The advantage of volatility, or standard deviation, is that it has the same dimension as the variable under investigation, whereas the dimension of variance would be the square of that dimension.

As we show in the appendix, the volatility decomposition is simple and its key formula can be derived with less effort than that found in Fields [2003]. It turns out that the *absolute* risk contribution of each factor is equal to the factor loading times the factor volatility, the σ , times its correlation, the ρ ,

with the independent variable. Using the notation in Equation 7, we find

$$\begin{aligned} \text{Absolute Volatility Contribution Factor } j \\ = \lambda_j \cdot \sigma(F_j^*) \cdot \rho(F_j^*, FR_1) \end{aligned} \quad (9)$$

where λ_j equals $(\gamma_1 - FR_0)$ for $j = 1$ and equals γ_j for $j = 2, \dots, k$. The relative or percentage risk contribution is then

$$\begin{aligned} \text{Relative Volatility Contribution Factor } j \\ = \frac{\lambda_j \cdot \sigma(F_j^*) \cdot \rho(F_j^*, FR_1)}{\sigma(FR_1)} \times 100\% \end{aligned} \quad (10)$$

CASE STUDY: SETTINGS

We now discuss practical issues such as the estimation of the factor exposures using forward-looking Monte Carlo simulations to estimate the risk factor contributions to future volatility. We also propose a sensible set of factors to use with DB pension funds and discuss the application of the framework to a typical pension fund.

Forward-Looking Mindset

We use a forward-looking mindset to provide insight into which risk factors are expected to contribute the most to the future funding-ratio risk. A risk decomposition based on historical time-series data would be too rigid in terms of conditioning the analysis on the recent past behavior of financial markets as well as on the current position of the pension fund (its funding ratio and investment strategy). The forward-looking simulation model allows for much greater flexibility, accommodating today's interest curves, funding ratio, investment strategy (including, if any, dynamic allocation rules), projected liability cash flows, and outlook on the financial markets.

The simulation model is used to create hypothetical equally probable future scenarios describing the financial asset returns, the pension fund's balance sheet items and the factors used in the risk decomposition. These scenarios allow us to compute the needed factor loadings, volatilities, and correlations. This forward-looking approach is cross-sectional—that is, based on simulated data across multiple forward-looking scenarios rather than based on historical time series in one single historical scenario.

Simulation Model

Our proprietary forward-looking Monte Carlo simulation model is based on two integrated submodels.⁷ The first is an economic scenario generator (ESG). Given user-defined inputs regarding initial values, expectations, and uncertainties around the expectations, the ESG simulates equally probable future scenarios for interest rate curves, inflation, asset class returns, and the required risk factors. Three blocks of variables are simulated:

1. *Economic state variables* such as CPI-inflation and various yield curves (nominal and real government bond curves, swap curve, and AA corporate bond curve). Yield curves are simulated via a limited number of factors describing the full curve. For instance, the real and nominal government curves rest on the Diebold and Li [2006] factorization in which the factors are level, slope, and curvature.
2. *Asset classes/benchmarks* such as Euro government bonds, high-yield bonds, European real estate, developed world equities and commodities covering 29 benchmarks.⁸
3. *Specific investment products/instruments* such as cash, customized portfolios of bonds with specific maturity constraints, and some actively managed products.

The second submodel is a policy simulator (PS), a straightforward mechanical model that combines the ESG data with the pension fund data. The PS starts with the current data for the pension fund, including liability value, total asset value, future liability cash flows, and investment strategy. These are combined with the ESG outcomes. In the case study, the investment horizon is one year. For each ESG scenario, forecasts of the value of the pension fund's assets, liabilities, and funding ratio one year on are generated by the PS. All these simulated values form the starting point for further statistical analyses to investigate the expected behavior of a given investment strategy.

ABC'S MODEL PORTFOLIO

This example is based on stylized data from an actual Dutch DB pension fund, which we have decided to name here "ABC." This fund is typical of other funds.

Its long-term strategic investment plan is expressed in terms of a model portfolio. This exercise was performed shortly after the management team completed its annual review proposing changes to the model portfolio, which is still waiting for the board's approval. The management team was interested in the implications of the changes in allocation on the expected funding-ratio risk over one year.

ABC follows a top-down investment process that disregards some practical issues—for example, active versus passive implementation decisions. Asset classes are represented by traditional benchmark indices.

At the top of the allocation pyramid, a distinction is made between the performance-seeking portfolio (PSP) and the LHP. A zero-investment interest rate swap overlay is part of the LHP. We assume that the swap portfolio perfectly tracks the relative changes in the liabilities, except for an adjustment for the cost of leverage. The role of the PSP portfolio is to generate additional returns in excess of the LHP over the long term. It is constructed on the basis of ABC's longer-term outlook. In theory, it should be a well-diversified portfolio maximizing the Sharpe ratio. In practice, investors may be satisfied with less. For simplicity, we disregard rebalancing within and between the LHP and PSP over the one-year simulation period.

Liability outflows and contribution inflows to the pension fund by the sponsoring company and its employees are assumed to take place at an annual frequency. We assume that such flows have just occurred and are reflected in the current values for the assets and liabilities. We further assume that the liabilities are nominal and that the regulator demands that the current zero-coupon swap curve must be used for determining their present value. The projected cash flows of the liabilities were provided by the pension fund and used to calculate their present value.

The pension fund implements a *targeted* IRH via delta hedging on the basis of *money duration* (i.e., modified duration, divided by 100, times the invested money amount). For instance, if the IRH is 70%, the LHP is constructed so that its money duration is equal to 70% of the money duration of the liabilities. Given the IRH, there are many possible strategies to implement it. One could implement it fully with the swap overlay, and then all the pension fund assets would be fully invested in the PSP. Pension funds do not usually go this far. Some of the assets are used in the LHP, which is then filled with duration-sensitive assets. In ABC's case, only government bonds

EXHIBIT 3

Overview of Current Balance Sheet and Model Portfolio

Panel A: Balance Sheet		Panel B: LHP (targeted IRH = 70.0%)				Panel C: PSP	
	Current Value		Allocation	Duration	Money Duration		Allocation
Assets		Bonds					
LHP	605 m EUR	Government	30.0%	6.9	22.8 m EUR	Cash	1.5%
PSP	495 m EUR + 1,100 m EUR	Corporate	25.0%	6.1	16.8 m EUR	High Yield	1.5%
						Leveraged Loans	2.5%
		Swap Overlay				Bonds_Emerging	4.0%
Liabilities	1,000 m EUR	Long Leg	44.7%	19.0	93.4 m EUR	Property	6.0%
		Short Leg	-44.7%	0.0		Equities_Developed	20.0%
Funding Ratio	110.0%					Equities_Emerging	5.0%
						Private Equity	2.5%
						Infrastructure	2.0%
		Total	55.0%		133.0 m EUR	Total	45.0%

Note: The asset classes in panels B and C are defined by the following euro-based references:

Government = CGBI WGBI All Maturities Index for the Netherlands; Corporate = CGBI Eurobig Corporates Index; Cash = One-month Euribor; High Yield = Merrill Lynch Global High Yield Index (hedged into euros); Leveraged Loans = S&P European Leveraged Loans Index; Bonds_Emerging = JPM EMBI Global Composite; Property = GPR 250 PSI Europe Index; Equities_Developed = MSCI World Index; Equities_Emerging = MSCI Emerging Markets Index; Private Equity = Private Equity Index of Cambridge Associates; Infrastructure = Proprietary Index.

and corporate bonds are used. Typically, the required money duration cannot be achieved with these physical LHP assets alone and thus the swap overlay is still needed. The PSP can contain interest rate-sensitive assets, for example, high-yield bonds. However, the pension fund's current practice is to ignore the interest rate sensitivity of the PSP. The IRH target is thus only implemented via the LHP. This is an important choice and we will come back to it later when discussing the risk analysis.

In the case study, the present date is the end of June 2014, and the initial key figures for pension fund ABC are summarized in Exhibit 3. Panel A shows the initial balance sheet numbers. The present value of liabilities is EUR 1,000 million. The assets amount to EUR 1,100 million, with over EUR 605 million (55%) allocated to the LHP and EUR 495 million (45%) to the PSP. The initial funding ratio is 110%.

Panels B and C provide further information on the LHP and PSP. The allocations are given in terms of a percentage of the total asset value. The LHP's targeted interest rate hedge is 70%. The liabilities, and the perfect swap, have a modified duration of 19 years. The modified durations of the physical IRH assets are taken from the index providers and amount to 6.9 for government bonds and 6.1 for corporate bonds. The targeted money duration of the LHP should be $0.7 \times (19/100) \times 1,000 = \text{EUR } 133 \text{ million}$. In order to get the EUR

133 million targeted money duration, the swap overlay legs must have values of 44.7% of EUR 1,100 million.

The PSP is risky: 87.8% of the PSP is allocated to high-risk assets (equities, property, emerging bonds, private equity, and infrastructure) and only 8.9% to medium-risk assets (high-yield and leveraged loans). Finally, there is a small allocation of 3.3% to cash.

ABC's One-Year ESG Forecasts

ABC's management team formulated a number of one-year forward-looking assumptions for the ESG dealing with expectations and standard deviations:

- For all curves, the starting point is their current shape. The various interest rate curves are stochastic, but expected to stay unchanged over one year. The volatility of changes in the government curve is fully implied by the ESG. The volatilities of swap and credit yield spreads are set in accordance with long-run data observations.
- For the 12-month inflation level, both the expectation and standard deviation are set at 1% for the coming year.
- For the liabilities, the future one-year returns are fully implied by the given liabilities structure and the swap curve simulations. The maturity of each

future cash flow is reduced by one month when we move one month forward in simulation time.

- For the government bonds, the future one-year return expectations and standard deviations are also fully implied by the curve simulations and the underlying simulation models that translate the nominal government curve simulations into simulated bond-market returns.
- Corporate bonds and each PSP asset are assigned a given one-year return expectation and standard deviation set consistent with the other assumptions.

Selected Factors

Our first factor, the return on liabilities (R_L), measures the risk impact of the IRH mismatch. We still need to define the other factors. There is a large breadth of papers on factors. Koedijk, Slager, and Stork [2016a, 2016b] reviewed this literature. Here, we add two remarks. First, many papers focus on risk factors within an asset class—for example, value, size, or momentum in equity markets. These are cross-sectional factors that can be used to reweight the market-capitalization benchmark constituents in smarter ways so as to gain exposure to the factors and potentially generate higher risk-adjusted returns. Such papers deal with active management or with how to deviate from the market-capitalization index. By construction, such factors tend to be weakly correlated with the asset class benchmark itself. Therefore, they are of little use in our case study in which active management within the asset classes is disregarded.

Second, we need, first and foremost, factors that are insightful for multiasset portfolios. We have already remarked that there is as yet no standard factor model for such portfolios (see endnote 2). We decided to be pragmatic and followed an approach similar to that proposed by Ransenberg, Hodges, and Hunt [2012]. To our knowledge, this is one of few studies to consider a factor-based risk decomposition of a pension fund, albeit based on surplus risk rather than funding-ratio risk. Ransenberg, Hodges, and Hunt used four economic factors capturing (1) real interest rate risk, (2) inflation risk, (3) economic growth risk: credits, and (4) economic growth risk: equities.⁹ Unlike us, they did not follow a forward-looking simulation approach, but worked with historical time series. Their factors were defined as the net returns to long–short portfolios of asset classes: (1) inflation-linked Treasuries minus one-month T-bills,

(2) nominal Treasuries minus inflation-linked Treasuries, (3) corporate bonds minus nominal Treasuries, and (4) equities minus one-month T-bills.

We moved away from Ransenberg, Hodges, and Hunt proxies for factors where appropriate. We had a preference not only for nonoverlapping factors but also for pure economic risk factors such as interest rates, yield spreads, and inflation, as opposed to total or excess returns of asset classes. In our view, pure factors make it easier to understand the pure drivers of individual asset classes and portfolios invested in them. Nevertheless, our factor definitions had to accommodate the constraints imposed by the Monte Carlo simulation model, and it was not always possible to escape using excess returns.

If we look at, for instance, the real interest rate risk factor—the first factor of Ransenberg, Hodges, and Hunt—their proxy, the excess return of inflation-linked Treasuries, is not only driven by real interest rate risks but also by actual inflation, thus overlapping with inflation risk, their second factor. We prefer to reduce the overlap to a minimum when possible. This explains the differences in choices for real interest rate risk and inflation risk.

Our choice of proxies for the credit element of economic growth risk includes both high-yield and investment-grade credit risk. Ideally, we would have preferred to use equivalent definitions in terms of spreads for both, but the lack of data for high-yield bonds, which is required to build a reliable model for Monte Carlo simulations, forced us to fall back on the excess return approach to proxy high-yield credit risk.

Additionally, we prefer to strip the other risks out of the equity risk, and for this reason we opted to use the perhaps less-conventional excess return of equities over high-yield bonds in order to capture the pure equity element of economic growth risk.

Finally, we can think of our choice of proxies for risk factors as a risk ladder in which each higher rung adds an extra pure risk factor. We begin with real-rate risk. Then we add two inflation risks, partially overlapping one another, of which one takes us to inflation-linked government bonds and the other to nominal government bonds. Next is credit risk, divided into two rungs, bringing us first to investment-grade credits and then to high-yield credits. The last rung adds pure equity risk. More precisely, the definitions are as follows:

- *Real-Rate Risk*: the 1-year change (first difference) in the 10-year real zero-coupon government bond yield

EXHIBIT 4

Balance Sheet and Model Portfolio: One-Year-Ahead Expectations and Standard Deviations for Returns and Funding-Ratio Level

Panel A: Balance Sheet			Panel B: LHP			Panel C: PSP		
	Expect.	St. Dev.		Expect.	St. Dev.		Expect.	St. Dev.
Assets	5.21%	8.66%	Bonds			Cash	0.29%	0.07%
LHP	4.26%	10.07%	Government	1.85%	3.00%	High Yield	4.30%	10.00%
PSP	6.37%	13.20%	Corporate	2.40%	3.20%	Leveraged Loans	4.00%	7.50%
						Bonds_Emerging	4.60%	13.00%
Liabilities	2.95%	9.19%	Swap Overlay	2.66%	9.20%	Property	5.60%	15.00%
						Equities_Developed	7.00%	17.00%
Funding Ratio						Equities_Emerging	8.70%	25.00%
Initial Level	110.00%	/				Private Equity	8.60%	16.00%
One-Year Level	112.72%	7.31%				Infrastructure	6.30%	15.00%
Return	2.47%	6.65%						

- *Inflation Risk*: uses two factors, one forward looking and one backward looking:
 - *Actual Inflation Risk*: the 1-year rate of change of the consumer price index
 - *Break Even Inflation (BEI) Risk*: the 1-year change in the 10-year BEI rate
- *Economic Growth–Credit Risk*: also considers two factors to distinguish investment-grade from high-yield credit risk:
 - *IG Credit Risk*: the 1-year change in the spread between the 10-year AA corporate credit yield and nominal zero-coupon government bond yield
 - *HY Credit Risk*: the 1-year excess return of high-yield bonds versus investment-grade corporate bonds
- *Economic Growth–Equity Risk*: the one-year excess return of an equity composite (80% developed and 20% emerging equities)¹⁰ versus high-yield bonds

We chose the 10-year maturity for the curve-related factors for pragmatic reasons. This lies somewhat in between the duration of the physical hedging assets and the duration of liabilities. Furthermore, 10-year yields are often used by fixed-income investors as a representative benchmark yield. The various benchmarks underlying the simulations of the last two long–short factors can be found in the notes under Exhibit 3.

CASE STUDY: RISK ANALYSIS

We now propose the use of a two-step OLS approach for estimating the factor exposures. The exposure to the mismatch factor is estimated first and the other factor exposures are estimated in a second step using a constrained regression. We also highlight the differences between assets-centric and LHP-centric risk decomposition, explaining our preference for the assets-centric approach. We end with a look through the IRH mismatch risk.

Recap of the Methodology

Our risk decomposition relies on the factor model given in Equation 7. With $k = 7$ and using the observations generated by 10,000 simulation scenarios, we first estimate the betas in Equation 4 via OLS. We then calculate the factor loadings in Equation 7, followed by calculating (1) the residuals in Equation 7, (2) the standard deviations of the rescaled factor returns and of the residuals, and (3) the correlation coefficients of the rescaled factor returns and of the residuals with the simulated one-year-ahead funding ratios. From Equations 9 and 10, we can then calculate the absolute and relative volatility decompositions.

Model Portfolio: Expectations and Volatilities

Exhibit 4 shows the simulation results, with ABC's one-year-ahead expectations and standard deviations

for items in the balance sheet and their underlying building blocks.

The essential point in Panel A is that the funding ratio is expected to improve by 2.72 percentage points, from an initial 110.00% to 112.72%, but with a volatility of 7.31%, which indicates a relatively significant downside risk.

In the Netherlands, a crude rule of thumb is that the funding ratio needs to be about 125% to 130% to have a sound funding status, because at such levels the fund is likely to pay its liabilities even after accounting for increase in wages or costs of living. A funding ratio of 105% is considered by the Dutch regulator as the absolute minimal value for not being underfunded.¹¹ In this context, pension fund ABC starts in the gray in-between area and is expected to remain there.

Model Portfolio: Unconstrained Volatility Decomposition

Panel A in Exhibit 5 shows the results of the funding-ratio decomposition without constraining the OLS regression. Column 1 shows the estimated factor loadings in Equation 7. *T*-values are given in Column 2. Their absolute values are not overly relevant because they are directly impacted by the number of simulation scenarios generated, which, in our case, at 10,000 is high. The *t*-values are, however, informative in the relative sense when comparing them with one another. Columns 3 and 4 show the factor volatility of each factor and their correlation with FR_t . Finally, Columns 5 and 6 show the risk decomposition in the absolute and relative senses.

The factor model explains 95% of the expected 7.31% funding-ratio volatility. Three factors—equity risk, IRH mismatch risk, and HY credit risk—stand out contributing by 20% or more. Three other factors—real-rate risk, BEI risk, and IG credit risk—have more moderate and negative contributions. Actual inflation risk is absent.

The loading for the IRH mismatch factor is strongly negative, as seen in Exhibit 5, Panel A. With a targeted IRH of 70%, we would have expected a factor loading of about 0.70 minus the current funding ratio of 1.10—that is about -0.40 . However, we find a value of -0.66 , suggesting that there may be something wrong with the interest rate hedging. We come back to this in the next subsection.

For the moment, we focus on the combined role of three factors in Exhibit 5, Panel A: the real-rate risk, BEI risk, and IG credit risk factors. All three deal with 1-year-ahead changes in 10-year rates—namely in the 10-year real rate, the 10-year BEI rate, and the 10-year credit spread (AA corporate bonds minus nominal government bonds), respectively. If we add these changes, we get the 1-year change in the 10-year AA corporate bond yield. If we translate the factor loadings into (relative) weights, we still get something close to the 10-year credit yield change because the weights for the factor loadings are approximately 0.4, 0.4, and 0.2, not far from equal weights.

Our point is that these three factors must closely correlate with the 1-year change in the 10-year swap rate, a crucial conclusion. The first factor, the IRH mismatch risk, is mainly driven by swap rate changes. Because swap rate changes along the curve are highly correlated, we can expect a high correlation between the IRH mismatch risk factor and the other three factors. Moreover, the correlations should be negative because the first factor is a return factor whereas the other three deal with changes in yields or yield spreads. The simulation results confirmed this. In terms of the unrescaled factors in Equation 4, the correlation of the Mismatch Risk Factor, R_L in Equation 4, with the factors for real-rate risk, BEI risk and IG credit risk is -24% , -49% , and -81% , respectively.

All this boils down to the conclusion that we have multicollinearity explaining not only the highly negative loading on the IRH mismatch risk factor, but also the large negative loadings on the real-rate, BEI, and IG credit risk factors in Panel A of Exhibit 5. At 40%, the IRH mismatch risk relative contribution is high, but part of it is diversified away by the other three risk factors, which together have a negative relative contribution of -19% . This could suggest that the total IRH mismatch risk contribution would be around 20%, rather than 40%. Other possible effects may play a role here—for example, the model portfolio's LHP is not perfectly aligned with the liabilities return because of its government and corporate bonds. All in all, the results in Panel A are biased because of multicollinearity.

Model Portfolio: Constrained Volatility Decomposition

To cope with multicollinearity, we constrained the OLS estimation of Equation 4. This can be

EXHIBIT 5

Model Portfolio: Volatility Decompositions for the One-Year-Ahead Funding Ratio

	Factor Loading (1)	T-value (2)	Factor Vol. (3)	Correlation with FR_1 (4)	Absolute Risk (5) = (1)*(3)*(4)	Relative Risk (6)
Panel A: Based on Unconstrained Regression						
IRH MisMatch Risk	-0.66	-135.00	0.0930	-0.48	0.0293	40.03%
Real-Rate Risk	-5.36	-60.74	0.0044	0.10	-0.0025	-3.37%
Actual Inflation Risk	0.19	9.12	0.0098	0.04	0.0001	0.09%
BEI Risk	-5.19	-58.17	0.0050	0.23	-0.0061	-8.28%
IG Credit Risk	-2.84	-49.42	0.0055	0.35	-0.0054	-7.41%
HY Credit Risk	0.46	254.69	0.0865	0.42	0.0169	23.10%
Equity Risk	0.34	386.32	0.1670	0.65	0.0370	50.61%
Unexplained Risk	1.00	/	0.0144	0.26	0.0038	5.22%
Total					0.0731	100.00%
Panel B: Based on LHP-Centric Constrained Regression Assuming Effective IRH = 0.66 (66%)						
IRH MisMatch Risk	-0.44	/	0.0930	-0.48	0.0196	26.75%
Real-Rate Risk	-2.41	-37.04	0.0044	0.10	-0.0011	-1.52%
Actual Inflation Risk	0.13	5.95	0.0098	0.04	0.0000	0.07%
BEI Risk	-2.00	-33.47	0.0050	0.23	-0.0023	-3.20%
IG Credit Risk	-1.03	-22.75	0.0055	0.35	-0.0020	-2.67%
HY Credit Risk	0.47	236.87	0.0865	0.42	0.0172	23.48%
Equity Risk	0.34	349.81	0.1670	0.65	0.0366	50.08%
Unexplained Risk	1.00	/	0.0160	0.32	0.0051	7.02%
Total					0.0731	100.00%
Panel C: Based on Assets-Centric Constrained Regression Assuming Effective IRH = 0.75 (75%)						
IRH MisMatch Risk	-0.35	/	0.0930	-0.48	0.0154	21.03%
Real-Rate Risk	-1.14	-16.13	0.0044	0.10	-0.0005	-0.72%
Actual Inflation Risk	0.11	4.54	0.0098	0.04	0.0000	0.06%
BEI Risk	-0.63	-9.75	0.0050	0.23	-0.0007	-1.01%
IG Credit Risk	-0.24	-4.99	0.0055	0.35	-0.0005	-0.64%
HY Credit Risk	0.47	219.82	0.0865	0.42	0.0173	23.64%
Equity Risk	0.34	320.93	0.1670	0.65	0.0365	49.84%
Unexplained Risk	1.00	/	0.0174	0.33	0.0057	7.79%
Total					0.0731	100.00%

Notes: In each panel, the reported *t*-values are identical to those usually provided for the coefficient estimates when relying on OLS regressions via Equation 4. There is one exception, though, that pertains to the first factor in the unconstrained regression (Panel A). Testing there that the null hypothesis that the loading on the IRH mismatch risk factor is zero is, namely, the same as testing that in Equation 4 the coefficient β_1 equals 1 (not zero). The reason for this is that, under reference to Equation 7, the factor loading on the IRH mismatch risk factor is defined as $FR_0 \cdot \beta_1 - FR_0$, which is only zero if β_1 is 1, leaving aside the trivial, but unrealistic, limiting case when $FR_0 = 0$.

achieved using information already available beforehand. For instance, the target IRH is one of the key decisions; and if we assume it is accurately implemented, we could—see Equation 8—constrain the loading $\gamma_1 - FR_0$ in Equation 7 to $0.70 - FR_0 = 0.70 - 1.10 = -0.40$. However, we can better use the simulation data to see how effective the hedge has really been. We reiterate that the PSP was ignored when implementing

the IRH. We can use a regression to see how effective the IRH of the LHP has been in the 10,000 simulated scenarios. The OLS outcomes below contain the answer (*t*-values in parentheses):

$$\Delta LHP = 6.36 + 0.66 \times \Delta LIABILITIES + residual \quad (11)$$

(80.1) (799.8)

where $R^2 = 98\%$, and ΔLHP and $\Delta LIABILITIES$ represent changes between time zero and one year, in millions of euros in the market value of the LHP and the liabilities, respectively. The 0.66 means that changes in the liabilities' market value are on average hedged for 66% by changes in the LHP's market value. Here, we call the coefficient 0.66 the *LHP-centric* effective IRH. Interestingly, with an R-squared of 98%, ΔLHP and $\Delta LIABILITIES$ are highly correlated. The delta-hedging process does a good job, although there is a small downward bias in the effective slope coefficient when compared to its targeted size. This may arise because (1) convexity effects are ignored, or (2) yield changes are not perfectly parallel along the curve, or (3) durations of the physical hedging assets given by index providers are not based on the swap curve, which should have been used here because ABC's liabilities are valued against this.

Although 0.66 for the LHP-centric effective IRH is compliant with ABC's IRH implementation process, it is certainly not the most accurate estimate of ABC's effective IRH. The PSP returns are also likely to be correlated with the liability returns, albeit to a much lesser extent. If we also consider the PSP, then the analysis is *assets-centric* and the equivalent to Equation 11 becomes

$$\Delta ASSETS = 35.1 + 0.75 \times \Delta LIABILITIES + residual \quad (12)$$

(51.0) (105.5)

where $R^2 = 53\%$, and so the assets-centric effective IRH is 0.75. Unlike the LHP, the total assets portfolio (i.e., the LHP together with the PSP) is not underhedging ABC's target by 4%. Instead, it is overhedging the target by 5%.

The alternative to the unconstrained risk decomposition is thus to first find the effective IRH via one initial regression of the type Equation 11 or Equation 12. This effective IRH must still be divided by FR_0 to get an estimate for β_1 in Equation 4, which is then estimated via constrained OLS. The rest of the decomposition methodology is as previously stated.

In Exhibit 5, Panels B and C show the factor decomposition based on the constrained procedure. Panel B shows the LHP-centric decomposition and Panel C shows the assets-centric decomposition. The reported loadings for the mismatch factor are now $0.66 - 1.10 = -0.44$ and $0.75 - 1.10 = -0.35$. These decompositions differ from the unconstrained decomposition in Panel A. The differences are well in line with the previous discussion on multicollinearity.

For the Actual Inflation Risk, HY Credit Risk, and equity risk factors, the differences between the various relative risk contributions are negligible. The relative IRH mismatch risk component falls from 40% (Panel A) to 27% (Panel B) to 21% (Panel C). This parallels similar falls in the diversifying power of the components for the Real Rates factor (from -3.4% to -1.5% to -0.7%), the BEI factor (from -8.3% to -3.2% to -1.0%) and the IG credit factor (from -7.4% to -2.7% to -0.6%). Finally, it is interesting to note that the unexplained risk is larger in Panel C than in A, which is not surprising when going from unconstrained to constrained analysis. Although the assets-centric decomposition was not affected by the biases that popped up in the unconstrained analysis, some still appeared in the LHP-centric decomposition. Thus, the assets-centric approach provides the most accurate risk decomposition.

IRH Mismatch Risk: A Look Through

After filtering out the precise IRH mismatch risk, we can still take the analysis one step further. The IRH mismatch risk is driven by the liabilities return R_L . In turn, in our case study, the volatility of the latter is fully driven by the swap curve dynamics. This link can be made more explicit. It is not difficult to look through R_L in terms of swap curve yields. In fact, using the simulated scenarios, it appeared possible to approximate R_L with just one single-yield maturity point, with the best fit obtained with terms to maturity of about 20 years. Perhaps the most natural maturity point is 19 years because that coincides with the initial duration of ABC's liabilities.

Inspired by the relationship between the change in a bond's relative price, yield to maturity change, (Macaulay) duration, and convexity, we chose a specification that explains R_L from two related variables. Let the 19-year zero-coupon swap yield (expressed as a percentage) at time 0 and 1 year be given by $SY_{19,0}$ and $SY_{19,1}$, and let ΔSY_{19} denote their first difference; then, the specification and its estimation results are (t -values in parentheses)

$$R_L = 2.38 - 16.72 \times \frac{\Delta SY_{19}}{1 + 0.01 \times SY_{19,0}} + 173.80 \times \left[\frac{\Delta SY_{19}}{1 + 0.01 \times SY_{19,0}} \right]^2 + residual \quad (13)$$

(313.1) (-1316.5) (144.7)

where $R^2 = 99.5\%$. This equation means that the 19-year swap rate alone is sufficient to explain almost the full 21.3% relative IRH mismatch risk component in Panel C in Exhibit 5.¹²

When presenting the risk decomposition results such as those in Panel C of Exhibit 5, it is a matter of taste as to whether or not to look through R_L . If one does, then one only needs to substitute Equation 13, including the estimated coefficients, into the (constrained) factor model Equation 7 and take it from there. That opens the possibility for grouping factors together. For instance, in our case study, before making the substitution, one could first rewrite in Equation 13 the swap yield level/change in terms of the real rate, the BEI, and the swap spread levels/changes. In the total risk decomposition, one could then group together the two real-rate components and the two BEI components.

Finally, pension fund ABC discounts its liabilities with the zero swap curve. Pension funds and sponsors operating under a different regulatory framework could be required or allowed to discount liabilities differently—for example, using an AA corporate bond curve or using the expected return on the assets. In such cases, one needs to customize the look-through Equation 13, but the rest remains the same. In this sense, the risk framework we propose is universal.

CONCLUSION

To our knowledge, we are the first to propose a decomposition framework for funding-ratio risk. It is based on a factor model and is flexible with regards to the choice of factors. We illustrate the application of the framework using a real case study based on forward-looking simulations rather than historical regression, because these simulations are much more powerful and useful for pension funds. We use real-rate risk, inflation risk, and two economic growth risks (i.e., credit risk and equity risk) as risk factors. We also include a novel risk factor to accurately measure the impact of not hedging fully the interest rate sensitivity of liabilities. The case study shows that our decomposition framework can give valuable insights about the key risk exposures in a pension fund's investment strategy.

We used standard deviation, or volatility, as a risk measure. This is justifiable if the funding-ratio distribution is normal, a reasonable assumption in our case.¹³ When the normal distribution can no longer be

considered—for example, if nonlinear derivatives such as swaptions or equity options are used for downside protection—one could turn to the VaR methodology and decompose the chosen (simulated) VaR.¹⁴

APPENDIX

Decomposing Volatility

Any linear factor model can be written as $Y = \beta_0 + \beta_1 \cdot X_1 + \dots + \beta_{n-1} \cdot X_{n-1} + u$, where Y, X_i with $i \in \{1, \dots, k-1\}$ and u are respectively the dependent variable, independent variable i , and the residual term. The betas are the factor loadings. To simplify notation, we define u as X_k . Hence, we can write the full model as $Y = \beta'X$ where β and X are $(k \times 1)$ column vectors. We write the $(k \times k)$ covariance matrix of the X -variables as $cov(X)$ and the variance of Y as $var(Y)$. Volatilities are symbolized by σ . A result that is used below starts with the correlation between Y and X_j :

$$\rho_{Y, X_j} = \frac{cov(Y, X_j)}{\sigma_Y \cdot \sigma_{X_j}} = \frac{cov(\beta'X, X_j)}{\sigma_Y \cdot \sigma_{X_j}} = \frac{\sum_{i=1}^k cov(X_i, X_j) \cdot \beta_i}{\sigma_Y \cdot \sigma_{X_j}} \rightarrow \sum_{i=1}^k cov(X_i, X_j) \cdot \beta_i = \sigma_Y \cdot \sigma_{X_j} \cdot \rho_{Y, X_j} \quad (A-1)$$

The volatility of Y can be written as¹⁵

$$\sigma_Y = \frac{1}{\sigma_Y} \cdot var(Y) = \frac{1}{\sigma_Y} \cdot (\beta' \cdot cov(X) \cdot \beta) \quad (A-2)$$

Now, the expression $cov(X) \cdot \beta$ defines a $(k \times 1)$ column vector of which the j th row element is given by Equation A-1. Hence, Equation A-2 becomes

$$\sigma_Y = \frac{1}{\sigma_Y} \cdot \sum_{j=1}^k (\beta_j \cdot \sigma_Y \cdot \sigma_{X_j} \cdot \rho_{Y, X_j}) = \sum_{j=1}^k (\beta_j \cdot \sigma_{X_j} \cdot \rho_{Y, X_j}) \quad (A-3)$$

Equations 9 and 10 in the main text are derived from Equation A-3, which had already been derived earlier by Fields [2003], albeit in a different context than ours and following a more involved reasoning.

ENDNOTES

The views and opinions expressed herein are those of the authors and do not necessarily reflect the views of BNP Paribas Asset Management, its affiliates, or employees.

¹For example, new amendments, issued in 2011 and effective as of 2013, to IAS 19 Employee Benefits require (listed) companies to mark to market valuations of both the pension fund's assets and liabilities, leading to a greater volatility of the net pension position in the sponsor's balance sheet.

²Here are some examples: (a) Litterman and Scheinkman [1991] proposed a three-factor model for government bond returns, which is in spirit the same kind of model as the Diebold and Li [2006] factorization of the government yield curve discussed later in the article; (b) Fama and French [1993] extended their three-factor equity model with two bond factors to capture government and investment-grade corporate bonds and the commonalities in bond and stock returns; (c) Fung and Hsieh [2004] developed a seven-factor model for hedge funds; and (d) Della Corte, Riddiough, and Sarno [2014] recently suggested a two-factor model for currencies. At present, there is no generally accepted factor model for multiasset portfolios.

³By "model-free," we mean that the true structure of the examined data is not needed in advance. The only assumption is that volatilities and correlations exist and can be estimated with the data.

⁴In this article, returns are always expressed as discretely compounded rates of return.

⁵Leibowitz, Kogelman, and Bader [1994] and Swierstra [2011] overlooked an issue that occurs when assets and liabilities are both normally distributed. In such a case, their ratio follows the Cauchy distribution, which has undefined moments. However, as long as the outcomes in the numerator are not approaching zero, this issue is not likely to cause a concern. In our practical application with funding ratios, this issue is not of concern.

⁶The proof of Equation 8 requires showing that γ_1 must be equal to IRH. We start with the observation that an interest rate hedge of IRH means that given an absolute change in the value of liabilities, the assets are expected to change by an amount of IRH times the liabilities change. This is $E(R_A \cdot A_0 | R_L \cdot L_0) = \text{IRH} \cdot R_L \cdot L_0$, which can be rewritten as $E(R_A | R_L \cdot L_0) = \text{IRH} \cdot R_L \cdot L_0 / A_0 = (\text{IRH} / \text{FR}_0) \cdot R_L$. In Equation 4, this means that $\beta_1 = \text{IRH} / \text{FR}_0$; and so by using the definition of γ_1 , we find that $\gamma_1 = \text{FR}_0 \cdot \beta_1 = \text{FR}_0 \cdot (\text{IRH} / \text{FR}_0) = \text{IRH}$.

⁷The simulation model is developed and maintained by the financial engineering team at BNP Paribas Asset Management and is used by various portfolio management teams for strategic analyses, in particular in the areas of retirement and Solvency II solutions.

⁸These can be broken down into five benchmarks for government bonds, three for credits, two for hybrids (convertible bonds and emerging markets debt), three for property, nine for equity, and seven for alternatives. Of these, there

are still 13 benchmarks that are available in two currency hedging versions, namely unhedged and fully hedged.

⁹For simplicity we disregard here another factor that Ransenberg, Hodges, and Hunt [2012] used to account for actuarial uncertainties regarding future liability cash flows.

¹⁰For simplicity, we assumed that there was no rebalancing during the year.

¹¹The funding-ratio level of 105% still takes a safety cushion into account for unforeseen expenses and risks.

¹²The second independent variable in Equation 13 captures convexity effects. It does not have much incremental explanatory power. The first variable alone, measuring duration effects, already has an R-squared of 98.5%.

¹³The estimated skewness and excess-kurtosis coefficients for ABC's model portfolio are only 0.50 and 0.41, respectively.

¹⁴See Hallerbach's [2002] discussion of the VaR decomposition in a simulation context. However, the precise way to apply this in our approach is still subject to further research.

¹⁵Equation A-2 is fully consistent with the Euler decomposition of the standard deviation, $\sigma_Y = \beta' \cdot \frac{\delta \sigma_Y}{\delta \beta}$ where the k elements of $\frac{\delta \sigma_Y}{\delta \beta}$ are the marginal risk contributions of the X -variables.

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